Residual strength of a fiber reinforced metal matrix composite with a crack

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Abstract

The residual strength of a cracked unidirectional fiber reinforced metal matrix composite is studied. We propose a bridging model based on the Dugdale strip yielding zones in the matrix ahead of the crack tips that accounts for ductile deformations of the matrix and fiber debonding and pull-out in the strip yielding zone. The bridging model is used to study the fracture of an anisotropic material and its residual strength is calculated numerically. The predicted results for a SiC/titanium composite agree well with the existing experimental data. It is found that a higher fiber bridging stress and a larger fiber pull-out length significantly contribute to the composite's residual strength. The composite's strength may be more notch-insensitive than the corresponding matrix material's strength depending on several factors such as fiber-matrix interface properties and the ratio of the matrix modulus to an 'effective modulus' of the composite. © 1997 Elsevier Science B.V.

1. Introduction

The interest in fiber reinforced metal matrix composites (MMC's) is still growing because of their advantages including high toughness, and resistance to impact and thermal shock over ceramic matrix composites (CMC's) and polymer matrix composites (PMC's). Even though substantial progress has been made in understanding the strength of MMC's [1–10], satisfactory theoretical predictions of their strength in the presence of notches and cracks can not be made. However, this is essential for the design of aerospace and other engineering components. The notch strength behavior of MMC's is different from that of their matrices. There should also be significant differences between the notch strength behav-

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crack. The effects of bridging stress and fiber pull-out length on the residual strength are studied. The predicted strength of a unidirectionally SiC fiber reinforced titanium matrix composite with a central crack is found to agree well with the experimental data.

2. Governing equations

We consider plane stress elastic deformations of a unidirectionally fiber-reinforced metal matrix plate. Assuming that it can be regarded as orthotropic, equations governing its deformations are [12]

\[
\begin{align*}
\beta_1 \frac{\partial^2 \nu_1}{\partial y_1^2} + \frac{\partial^2 \nu_1}{\partial y_2^2} + \beta_2 \frac{\partial^2 \nu_2}{\partial y_1 \partial y_2} &= 0, \\
\frac{\partial^2 \nu_2}{\partial y_1^2} + \frac{\partial^2 \nu_1}{\partial y_2^2} + \beta_2 \frac{\partial^2 \nu_1}{\partial y_1 \partial y_2} &= 0,
\end{align*}
\]

(1)

where the transformed stresses \(\tau_{\alpha\beta}\) (subscripts \(\alpha\) and \(\beta\) take values 1 and 2) and stresses \(\sigma_{\alpha\beta}\) are related to the transformed displacements \(v_\alpha\) by

\[
\begin{align*}
\tau_{11} = \frac{\sigma_{11}}{\lambda} &= \frac{E_0}{1 - \nu_0^2} \left( \frac{\partial v_1}{\partial y_1} + \nu_0 \frac{\partial v_2}{\partial y_2} \right), \\
\tau_{22} = \lambda \sigma_{22} &= \frac{E_0}{1 - \nu_0^2} \left( \frac{\partial v_2}{\partial y_2} + \nu_0 \frac{\partial v_1}{\partial y_1} \right), \\
\tau_{12} = \sigma_{12} &= \frac{E_0}{2(\kappa + \nu_0)} \left( \frac{\partial v_1}{\partial y_2} + \frac{\partial v_2}{\partial y_1} \right).
\end{align*}
\]

(2)

In Eqs. (1) and (2), the transformed coordinates \(x_\alpha\) and the transformed displacements \(v_\alpha\) are given by

\[
\begin{align*}
y_1 &= x_1/\sqrt{\lambda}, & y_2 &= x_2/\sqrt{\lambda}, \\
v_1 &= u_1\sqrt{\lambda}, & v_2 &= u_2/\sqrt{\lambda},
\end{align*}
\]

(3)

where \(x_\alpha\) are rectangular Cartesian coordinates and \(u_\alpha\) the displacements of a point. Various constants in Eqs. (1)–(4) are given by [12,13]

\[
\begin{align*}
\beta_1 &= 2(\kappa + \nu_0)/(1 - \nu_0^2), \\
\beta_2 &= \nu_0 \beta_1 + 1, \\
E_0 &= \sqrt{E_{11} E_{22}}, & \nu_0 &= \sqrt{\nu_{12} \nu_{21}}, \\
\lambda &= (E_{11}/E_{22})^{1/4}, & \kappa &= E_0/(2\mu_{12}) - \nu_0,
\end{align*}
\]

(6)

where \(E_{11}, E_{22}, \mu_{12}, \nu_{12}\) and \(\nu_{21}\) are elastic constants of the orthotropic plate. For a unidirectionally reinforced MMC with fibers in the \(x_1\)-direction [14–16],

\[
\begin{align*}
E_{11} &= V_t E_t + (1 - V_t) E_m, \\
E_{22} &= \frac{1 + 2\eta V_t}{1 - \eta V_t} E_m, & \eta &= \frac{E_t/E_m - 1}{E_t/E_m + 2}, \\
\mu_{12} &= \frac{(1 + V_t) \mu_t + (1 - V_t) \mu_m}{(1 - V_t) \mu_t + (1 + V_t) \mu_m}, \\
\nu_{12} &= \frac{E_{11}}{E_{22}} \nu_{21} = V_t \nu_t + (1 - V_t) \nu_m,
\end{align*}
\]

(7)

where \(V_t\) is the fiber volume fraction and subscripts \(f\) and \(m\) stand for the fiber and the matrix, respectively.

3. The proposed model and the problem formulation

Consider an infinite unidirectionally fiber reinforced MMC plate with a through crack of length \(2a_0\) perpendicular to the fiber direction. The plate is subjected to a uniform tension \(\sigma_c\) at infinity in the fiber \((x_1)\)-direction.

For a SiC fiber reinforced titanium alloy \((\text{Ti–6Al–4V})\), Connell et al. [10] observed a narrow plastic strip ahead of the crack tip and essentially all fibers within the plastic zone broke prior to the catastrophic fracture of the plate. Based on the experimental observations, they proposed a two level rectilinear bridging law, i.e. the plastic strip is treated as a bridged crack and the crack bridging stress is governed by the unnotched composite’s strength during the first stage and by the matrix yield stress with a contribution from fiber pull-out during the second stage following fiber breakage. The bridging is lost when the local strain reaches the failure strain of the matrix. This model does not consider fiber debonding, frictional slip and pull-out processes which will definitely influence the bridging law, a relationship between the bridging stress and the separation of the bridged crack faces. Also, fiber bridging is not necessarily lost in the wake of the matrix crack as the
broken fibers in the pull-out process may still bridge the matrix crack [11].

Here we propose a bridging model based on both matrix yielding and micromechanical analyses of fiber debonding and pull-out. It is assumed that both the matrix and fibers contribute to the bridging stress, i.e.

$$\sigma = \sigma_m V_m + \sigma_t V_t$$

where $\sigma_m = 1 - V_t$, $\sigma$ is the total bridging stress, $\sigma_m$ the stress due to matrix yielding and $\sigma_t$ the stress due to fiber debonding and pull-out. It is reasonable to assume that $\sigma_m$ is given by the Dugdale model [17]

$$\sigma_m = \sigma_y H(\delta_c - \delta)$$

where $\delta$ is the separation between the upper and lower surfaces of the yielding zone (faces of the bridged crack), $\sigma_y$ is the yield stress of the matrix, $\delta_c$ is the critical separation at which crack growth occurs in the matrix and $H(\cdot)$ is the Heaviside step function. $\delta_c$ is related to the yield stress and the critical value, $J_c$, of the $J$-integral by

$$\delta_c = J_c / \sigma_y$$

The bridging stress $\sigma_t$ due to fibers can be determined from micromechanics analyses of the stress transfer between fibers and matrix [18], fiber debonding [19–21] and fiber pull-out. As we replace the strip yielding zone by a bridged crack, the elastic analyses [18–20] may be validated in the elastic region at the bridged crack face and approximately apply to the debonding bridging in the real problem. The elastic analyses [19–21] have also been employed to study the fatigue crack growth [22,23]. In the first stage fibers are intact and are bonded perfectly to the matrix. When the shear stress at the interface between the fiber and the matrix reaches a critical value, the fiber is debonded from the matrix but still remains unbroken. Usually, the energy contribution from the first stage is negligibly small and we will not consider debonding during this stage. In fact, we will see below that the bridging energy of fiber debonding and slipping is also very small as compared with the fiber pull-out energy in MMC’s especially when the fiber-matrix interface is not too weak. In the debonding stage, $\sigma_t$ is related to the separation $\delta$ by [20,21]

$$\sigma_t / \sigma_0 = \frac{\sqrt{\delta / \delta_0}}{H(\delta_0 - \delta)}$$

(11)

where $\sigma_0$ is the fiber strength (here we assume that fibers have a deterministic strength),

$$\delta_0 = \frac{(1 - V_t)^3 E_m^2 \sigma_0^2}{2 \pi E_{11}^2 R}$$

(12)

$R$ is the fiber radius and $\tau$ the frictional shear stress at the interface between the fiber and the matrix. When the fiber stress reaches the fiber strength $\sigma_0$, fibers are broken and are subsequently pulled out. The pull-out stress may be obtained through a simple shear lag analysis.

$$\sigma_t / \sigma_c = \left[ 1 - (\delta_c - \delta_0) / (\delta_c - \delta_0) \right] \times H(\delta_c - \delta) H(\delta - \delta_0)$$

(13)

where $\delta_c$ is the fiber pull-out length and $\sigma_c$ a critical stress. Usually $\sigma_c$ is less than $\sigma_0$. It is expected that the stress in the fiber drops rather precipitously from $\sigma_0$ to $\sigma_c$ immediately after it breaks. The stress drop $\sigma_0 - \sigma_c$ is a characteristic of the fiber–matrix interface properties. A simple shear lag analysis gives

$$\sigma_c = \frac{2 \tau \delta_c}{R}$$

(14)

$\delta_c$ in Eqs. (13) and (14) is not a constant but varies from fiber to fiber. Here we assume that there exists a statistical average value of $\delta_c$ determined experimentally and use it below. As mentioned above, for MMC’s, the fiber debonding energy is generally negligibly small as compared with the fiber pull-out energy and $\delta_0 \ll \delta_c$. For example, for a SiC/titanium composite studied in [6,10], $\delta_0 = 0.1$ $\mu$m, the debonding energy $G_0 = 0.032$ kJ/m$^2$, the average fiber pull-out length exceeds 100 $\mu$m and the pull-out energy is a fraction of the bridging energy, 72 kJ/m$^2$. Hence, the contribution from fiber debonding may be ignored in the residual strength calculations. However, it is more reasonable to regard the debonding energy as the crack tip energy because the scale of this debonding bridging is usually very small as compared with other bridging scales and macro-crack lengths. The debonding bridging scale may be characterized by the parameter $E_0 \delta_0 / (V_t \sigma_0)$ [24], where $E_0$ is an ‘effective modu-
lus’ of the cracked orthotropic plate given below. For the SiC/titanium composite [6], this parameter equals about 36 \( \mu m \) which is an order of magnitude smaller than the macro-crack length in MMC’s. Hence, we assume that the bridging stress due to fibers is fully described by Eq. (13) with \( \delta_0 = 0 \) and a crack tip energy \( G_\delta = G_\theta \) exists. A schematic representation of the bridging law described by Eqs. (8), (9) and (13) is shown in Fig. 1.

Based on the above assumptions, the boundary conditions of the problem can be expressed as

\[
\begin{align*}
\sigma_{11} &= -\sigma_t + H(|x_2| - a_x)(1 - V_t)\sigma_s \\
&\quad + H(|x_2| - a_y)V_t\sigma_c(1 - \delta_0/\delta_c), \\
&\quad x_1 = 0, \quad |x_2| \leq a, \\
\sigma_{12} &= 0, \quad x_1 = 0, \quad |x_2| < \infty, \\
u_1 &= 0, \quad x_1 = 0, \quad |x_2| > a, \\
\end{align*}
\]

(15)–(17)

where \( a = a_x + \Delta a \) is half of the length of the initial crack plus the length of the yielding/bridging zone, \( a_x \) and \( a_y \) initially equal \( a_0 \) and become larger when complete fiber pull-out and/or real crack growth in the matrix occur.

The boundary value problem defined by Eqs. (1), (2), (15)–(17) with transformed variables (Eqs. (3) and (4)) results in the following singular integral equation:

\[
\begin{align*}
&\frac{1}{2\pi} \int_{-1}^{1} \frac{\phi(s) \, ds}{s - r} + H(|r| - r_0)V_t\left(\frac{a}{a_0}\right) \\
&\quad \times a_0^* \left(\frac{\sigma_c}{\sigma_s}\right) \int_{-1}^{r} \phi(s) \, ds \\
&= -\frac{\sigma_x}{E_0} + H(|r| - r_0)(1 - V_t)\frac{\sigma_s}{E_0} \\
&\quad + H(|r| - r_0)V_t\frac{\sigma_c}{E_0}, \quad |r| \leq 1,
\end{align*}
\]

(18)

where

\[
\phi(r) = \frac{\partial u_1(0, x_2)}{\partial x_2}
\]

(19)

is the dislocation density along the crack (including the bridged part) face, \( r = x_2/a \), \( r_0 = a_0/a \), \( r_y = a_y/a \),

\[E_0 = \frac{2\lambda E_0}{\sqrt{2(1 + \kappa)}}
\]

(20)

is an ‘effective Young’s modulus’ of the orthotropic material, i.e. the cracked orthotropic material behaves like an isotropic one with ‘Young’s modulus’ \( E_0 \) and

\[a_0^* = \frac{a_0}{E_0 \delta_0/\delta_c}
\]

(21)

is a nondimensional crack length. The separation displacement of the crack faces is related to \( \phi \) by

\[\delta = 2u_1(0, x_2) = 2a \int_{-1}^{r} \phi(s) \, ds.
\]

(22)

It is clear that \( \phi \) satisfies

\[\int_{-1}^{1} \phi(s) \, ds = 0.
\]

(23)

According to the singular integral equation method [25,26], Eq. (18) under the condition in Eq. (23) has a solution of the form

\[\phi(r) = \frac{\psi(r)}{\sqrt{1 - r^2}}, \quad |r| < 1,
\]

(24)

where \( \psi(r) \) is continuous and bounded on \([-1, 1]\).
The stress intensity factor at the tip of the bridged crack can be evaluated from

$$K_{\text{tip}} = -\frac{1}{2}E_0\sqrt{\pi a} \psi(1)$$

(25)

and the crack tip energy release rate is given by

$$G_{\text{tip}} = \frac{K_{\text{tip}}^2}{E_0}.$$  

(26)

4. Residual strength

For a given initial crack, Eq. (18) can be solved with increasing yielding/bridging length and the strength $\sigma_c$ can be evaluated from Eqs. (24)–(26) with $G_{\text{tip}} = G_c$. For a monolithic metal panel, the residual strength is reached when the opening displacement at the initial crack tip or the tail of the yielding strip equals $\delta_c = J_c / \sigma_c$. For the MMC, the residual strength is determined by choosing the maximum value of the applied stress $\sigma_0$ during the bridged crack growth.

It is convenient to write the solution of Eq. (18) as

$$\phi(r) = \frac{1}{E_0} \left[ \alpha_c \phi_1(r) + \sigma_c \phi_2(r) + \sigma_c \phi_3(r) \right]$$

(27)

where $\phi_i(r) (i = 1, 2, 3)$ is dimensionless and satisfies

$$\frac{1}{2\pi} \int_{-1}^{1} \frac{\phi_i(s) \ ds}{s - r}$$

$$+ H(|r| - r_d)\left(\frac{a_{\theta}}{a_0}\right)^2 \frac{\sigma_c / \sigma_0}{\delta_c / \sigma_0} \int_{-1}^{r} \phi_i(s) \ ds$$

$$= L_i(r), \quad |r| \leq 1,$$

(28)

and

$$\int_{-1}^{1} \phi_i(s) \ ds = 0,$$

(29)

where

$$L_1(r) = -1, \quad L_2(r) = H(|r| - r_y)(1 - V_t),$$

$$L_3(r) = H(|r| - r_d)V_t.$$  

(30)

Eq. (28) has a solution of the form

$$\phi_i(r) = \frac{\psi_i(r)}{\sqrt{1 - r^2}}, \quad i = 1, 2, 3,$$

(31)

where $\psi_i(r)$ is continuous and bounded on $[-1, 1]$. The stress intensity factor at the tip of the bridged crack can then be evaluated from

$$K_{\text{tip}} = -\frac{1}{2} \sqrt{\pi a} \left[ \sigma_c \psi_1(1) + \sigma_c \psi_2(1) + \sigma_c \psi_3(1) \right].$$

(32)

By equating $K_{\text{tip}}$ to the assumed critical effective stress intensity factor $K_c$ and noting that

$$K_c = \sqrt{E_0G_c}$$

(33)

where $G_c$ is the fiber debonding energy regarded as the crack tip energy, we obtain the following expression for the residual strength:

$$\frac{\sigma_R}{\sigma_c} = \max_{0 < a < a_0} \left[ \frac{1}{-\left(1/2\right)\psi(1)} \times \left[ 1 - V_t + \left(1/2\right)V_t \left(\frac{\sigma_c / \sigma_0}{\delta_c / \sigma_0} \frac{G_c}{G_c} \right)^{1/2} \right. \right.$$}

$$\left. \times \left[ \frac{1}{\sqrt{a / a_0}} - \frac{1}{2} \psi(1) \right] - \frac{\sigma_c}{\sigma_0} \left[ -\frac{1}{2} \psi(1) \right] \right]$$

(34)

Here

$$G_b = \sigma_c \delta_c \left[ (1 - V_t) + \frac{1}{2} V_t \frac{\sigma_c}{\sigma_0} \delta_c \right]$$

(35)

is the bridging energy.

5. Numerical results and discussion

We first consider a SiC/titanium composite studied in Refs. [6,10]. By equating the bridging energy $G_b$ in Eq. (35) to the experimental value 72 kJ/m² given in Ref. [10] and using the material properties data given in Ref. [6], we obtain $\sigma_c / \sigma_0 = 0.754$, $\delta_c / \delta_0 = 1.8125$ and $E_0 \delta_c / \sigma_0 = 14.5$ mm. The calculated residual strengths for half crack lengths $a_0 = 0.75, 1.5$ and $3.0$ mm are 894, 870 and 807 MPa, respectively, which are close to the experimental values [10] of 910, 850 and 810 MPa, respectively. The effects of the specimen size and strain hardening of the matrix are not considered. A finite specimen
will have lower residual strength than an infinite one, and strain hardening tends to increase the strength. Thermal residual stress effects are also not included in the present model. Though the effect is significant in determining the overall tensile behavior of MMC's \cite{27,28}, it may not strongly influence the residual strength of cracked MMC's.

Figs. 2–4 show the calculated normalized residual strength $\sigma_R^* = \sigma_R / \sigma_s$ versus the nondimensional crack length $a_0^* = a_0 / (E_0 \delta_s / \sigma_s)$ for various values of the bridging to yielding stress ratio $\sigma_c / \sigma_s$ and the critical displacement ratio $\delta_c / \delta_s$. The effective crack tip energy is neglected in the calculations since it is much smaller than the bridging energy as shown in Section 3. It can be seen from Figs. 2–4 that for fixed $\delta_c / \delta_s$ the residual strength increases with an increase in $\sigma_c / \sigma_s$. However, the material becomes more notch-sensitive for larger values of $\sigma_c / \sigma_s$, i.e. for large $\sigma_c / \sigma_s$, the residual strength decreases more rapidly with an increase in the initial crack length, especially for short cracks. For fixed $\sigma_c / \sigma_s$, the residual strength increases with an increase in $\delta_c / \delta_s$ and the composite's strength becomes less notch-sensitive. Fig. 5 shows the residual strength for a composite and for the corresponding metal matrix with $E_0 / E_m = 2.0$. The metal's residual strength is obtained from the Dugdale model

$$\sigma_R = \frac{2}{\pi} \cos^{-1} \left(e^{-\frac{E_m}{8 \sigma_m} \frac{\pi}{a_0}}\right).$$

We note that, for this case, the metal's strength is more notch-sensitive than the composite's except for very short cracks. However, the composite may be-
come more notch-sensitive than the matrix for lower values of $\delta_c/\delta_s$ and $E_0/E_m$. Hence, improved damage tolerance of MMC's may be achieved by increasing $\delta_c/\delta_s$ and $E_0/E_m$. A higher $\delta_c/\delta_s$ may be promoted by a weak fiber–matrix interface so that fibers break randomly in the plastic region giving a higher average pull-out length. However, a very weak interface will lower the fiber bridging stress resulting in a low residual strength of the composite. Hence, the fiber-matrix interface has to be optimized. We note that we have assumed a deterministic fiber strength. A statistical consideration of the fiber strength may result in a scattered residual strength distribution with a statistical average approximately described by the proposed model.

The model developed here for fiber reinforced MMC's may also be applied to particulate MMC's. For particulate MMC's, the critical displacement ratio $\delta_c/\delta_s$ is low as $\delta_s$ now represents the average particle size. The stress ratio $\sigma_c/\sigma_s$ is also low as $\sigma_c$ is proportional to $\delta_c$. Also, particle rupture often occurs in particulate MMC's. Hence, the particle bridging effect will not be significant. Fig. 6 shows the residual strength of a cracked particulate MMC. It is clear that particle bridging does not contribute much to the residual strength for cracks longer than

\[ 0.2E_0\delta_c/\sigma_s \quad (E_0 \text{ is the effective modulus of the particulate MMC}). \]

6. Concluding remarks

The crack bridging concept is used to study the residual strength of a cracked unidirectionally fiber-reinforced metal matrix composite. Dugdale strip yielding in the matrix and effects of fiber debonding and pull-out from the matrix are included in the analysis. The anisotropy of the material is considered through an effective modulus. The predicted results for a SiC/titanium composite agree well with the existing experimental data. It is found that a higher fiber bridging stress and a larger fiber pull-out length significantly contribute to the composite’s residual strength. The notch sensitivity of the composite’s strength depends upon several factors such as the fiber–matrix interface properties and the ratio of the matrix modulus to an ‘effective modulus’ of the composite.

References


