ADIABATIC SHEAR BANDS IN PLANE STRAIN DEFORMATIONS OF A WHA

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Abstract—We study plane strain thermomechanical deformations of a prismatic body with a rectangular cross-section and made of a tungsten heavy alloy (WHA). Tungsten and iron–nickel–tungsten (Fe–Ni–W) particles are modeled as thermally softening but strain and strain rate hardening. The deformations are assumed to be locally adiabatic and the effect of inertia forces is considered. The body is loaded by applying a normal component of velocity to the opposite edges; the speed increases from zero to the final value in 5 microseconds and is then kept steady there so that the maximum average strain rate is 5000 s⁻¹. Different volume percentages of Fe–Ni–W particles are taken to be randomly distributed in the cross-section. It is found that the time history of the compressive load required to deform the body is initially unaffected by the volume percentage of Fe–Ni–W particles. At an average strain of 0.10 these load histories begin to differ somewhat; at any given value of the nominal strain, the decrease in the magnitude of the load is not a monotonic function of the increase in the volume percentage of Fe–Ni–W particles. For a fixed volume percentage of Fe–Ni–W particles, different random distributions result in essentially the same load history but give different patterns of shear bands. For eight randomly distributed Fe–Ni–W particles clustered around the horizontal centroid axis, none of the shear bands passed through any one of these particles. © 1998 Elsevier Science Ltd. All rights reserved

I. INTRODUCTION

A tungsten heavy alloy (WHA) usually consists of 93% by weight tungsten particles interspersed in a matrix of iron–nickel or other materials. For its application as an armor material it is desirable to have as much tungsten, because of its high mass density, as possible. It is believed that the penetration performance of a WHA will be enhanced if adiabatic shear bands form in it continuously and the material fails along these bands (Magness and Farrand (1990)). An adiabatic shear band is a narrow region of intense plastic deformation that usually forms during high strain rate deformation of most metals and many polymers. A shear band is generally followed by a crack, and is an important failure mechanism in ductile materials. It is called adiabatic since there is not enough time for the heat to be conducted out of the hot and severely deformed region. The reader is referred to the book by Bai and Dodd (1992), a special issue of Applied Mechanics Reviews edited by Zbib et al. (1992), an issue of Mechanics of Materials edited by Armstrong et al. (1994), and the volume edited by Batra and Zbib (1994) for references on the subject.

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A penetration problem in general will involve three-dimensional deformations of the WHA; here we study, for simplicity, its plane strain deformations. The WHA is usually made by sintering and tungsten particles are interspread in the matrix phase. We assume that matrix particles are randomly distributed among the tungsten particles, the two sets of particles deform coherently in the sense that surface tractions and displacements among adjoining particles are the same. Thus no separation across the interface between two different material phases is allowed. The deformations are assumed to be locally adiabatic; this assumption appears reasonable because of the short times, of the order of a few microseconds, involved for the development of a shear band. Batra and Kim (1991), through numerical experiments, have shown that the effect of thermal conductivity on the time of initiation of a shear band is negligible; however, the post localization behavior is noticeably affected by heat conduction.

We investigate the effect of the volume of iron–nickel–tungsten (Fe–Ni–W) particles dispersed randomly among the tungsten particles and consider cases when the former equals 10, 8, 6, 4, 2, 0.2, 0.1, 0.02 and 0.01% of the total volume. For each one of the first five cases, seven different random distributions of Fe–Ni–W particles are analysed with the objectives of finding statistically the average strain at the initiation of a shear band and the standard deviation in this value. The computed results reveal that the initial distribution of Fe–Ni–W particles, provided it exceeds 0.1%, has a negligible effect on the time-history of the compressive load required to deform the body till the time a shear band initiates and also on the instant when a shear band initiates. For the present problem, it is not easy to decipher when a shear band initiates since it is difficult to formulate an objective criterion. The load required to deform the body drops noticeably much after (about 2 or 3 microseconds out of a total of 30 microseconds) the mesh has severely deformed; the mesh begins to deform intensely before that time. However, the number of bands, their locations and the general pattern of the deformed mesh depend upon the volume percentage and the distribution of Fe–Ni–W particles. Shear bands also form in pure tungsten and the load history in this case is quite different from that when there are Fe–Ni–W particles distributed randomly in tungsten particles. After one or more shear bands have developed in the block, dead zones with essentially zero strain rates and separated by these bands develop. We note that Batra and Peng (1995) studied a similar problem for tungsten and uranium blocks and also computed these dead zones. They considered a block of a homogenized tungsten alloy with 0.1, 0.2 and 0.3% randomly distributed defects, modeled as either very soft, very hard or 5% weak particles and employed the Johnson–Cook (1983) relation to simulate the thermoviscoplastic response of the material. Here we employ a similar constitutive relation except that the thermal softening is modeled by a power-law type function Zhou (1993) obtained by fitting a curve through the experimental data. We consider a two phase mixture with each constituent assigned material properties obtained from test data. Thus the speeds of elastic and plastic waves are different in the two constituents. We add that there is no failure or fracture criterion included in this work; thus each constituent can undergo unlimited amount of plastic deformations until it melts. A melted particle is modeled as a compressible ideal fluid. However, in our simulations, no material particle melted.

II. FORMULATION OF THE PROBLEM

We use rectangular Cartesian co-ordinates, with origin at the centroid of the rectangular 1 cm×2 cm block (cf. Fig. 1), to analyse plane strain, locally adiabatic,
thermomechanical deformations of a WHA consisting of tungsten and Fe–Ni–W particles. It is assumed that Fe–Ni–W particles are randomly distributed in the quarter cross-section lying in the first quadrant, they are situated symmetrically with respect to the horizontal and vertical centroidal axes in other quadrants, and during the deformation process, surface tractions and displacements at the interfaces between different particles are continuous. We use the referential or Lagrangian description of motion and refer the reader to Truesdell and Noll (1965) for equations expressing the balance of mass, linear momentum, moment of momentum and internal energy. We assume that the moment of momentum is identically satisfied and all of the plastic working rather than 90 to 95% of it as asserted by Taylor and Farren (1925) and Sulijoadikusumo and Dillon (1979) is converted into heating.

We assume that each material is isotropic, elastic-viscoplastic and obeys von Mises yield criterion with isotropic hardening; the radius of the yield surface at a material point increases with an increase in the effective plastic strain and effective plastic strain-rate and decreases with a rise in the temperature there. The flow stress, $\sigma_y$, at a material point is assumed to be given by

$$
\sigma_y = (A + B(\varepsilon_p)^n)(1 + C\ln(\dot{\varepsilon}_p/\dot{\varepsilon}_0))(1 - \beta\left(\left(\frac{T}{T_0}\right)^m - 1\right)).
$$

(1)

Here $A$, $B$, $n$, $C$, $\dot{\varepsilon}_0$, $\beta$ and $m$, are material parameters, $T_0$ is the room temperature in degrees Kelvin, $\varepsilon_p$ is the effective plastic strain, $\dot{\varepsilon}_p$ is time rate of change of $\varepsilon_p$ and $T$ is the present temperature in degrees Kelvin of the material particle. Equation (1) differs from the Johnson–Cook (1983) relation only in the dependence of the flow stress upon the temperature rise. Bell (1968) concluded from tests done in his laboratory that, for several metals, the yield stress is an affine function of the temperature rise. Zhou (1993) obtained

![Fig. 1. (a) A schematic sketch of the problem studied, and (b) the time history of the vertical component of velocity prescribed on the top and bottom edges.](image)
the temperature dependence indicated in eqn (1) by fitting a smooth curve to the experimental data for the Fe–Ni–W phase, and the homogenized WHA. In the absence of any test data for tungsten particles, he postulated a similar expression for tungsten and gave for it values of $\beta$ and $m$. These and values of $A$, $B$, $n$, $C$ and $\dot{\varepsilon}_0$ obtained by fitting curves to the data points computed from the relations given by Zhou (1993) for tungsten and Fe–Ni–W particles are listed below.

Tungsten:

\[
\begin{align*}
\rho &= 19300 \text{ kg m}^{-3}, \quad G = 155 \text{ GPa}, \quad A = 730 \text{ MPa}, \quad B = 562 \text{ MPa}, \\
n &= 0.0751, \quad C = 0.02878, \quad \dot{\varepsilon}_0 = 1.355 \times 10^{-7} \text{s}^{-1}, \quad c = 138 \text{ J kg}^{-1} \text{°C}^{-1} \\
m &= 0.15, \quad \beta = 2.4, \quad T_0 = 293 \text{K} \\
\end{align*}
\]

(2)

Fe–Ni–W:

\[
\begin{align*}
\rho &= 9200 \text{ kg m}^{-3}, \quad G = 98.84 \text{ GPa}, \quad A = 150 \text{ MPa}, \quad B = 546 \text{ MPa}, \\
n &= 0.208, \quad C = 0.0838, \quad \dot{\varepsilon}_0 = 6.67 \times 10^{-7} \text{s}^{-1}, \quad c = 382 \text{ J kg}^{-1} \text{°C}^{-1} \\
m &= 0.20, \quad \beta = 2.4, \quad T_0 = 293 \text{K}. \\
\end{align*}
\]

(3)

Here $\rho$ is the mass density, $G$ the shear modulus, and $c$ the specific heat of the material. We assume that the volumetric response of the material is elastic with the hydrostatic pressure $p$, taken to be positive in compression, related to the specific volume change by

\[
p = K(p/p_0 - 1) \tag{4}
\]

where $\rho_0$ equals the mass density in the reference configuration, $K = 317.45$ GPa for tungsten and 202.4 GPa for Fe–Ni–W particles. For the present problem, plastic deformations are expected to be dominant. Since plastic deformations are assumed to be independent of the hydrostatic pressure, the precise form of the equation of state will have a negligible effect on the time of initiation of a shear band. This was confirmed by running a few cases with the Mie–Gruniesen equation of state. The two sets of results differed only slightly in the magnitude of the compressive load required to deform the block, but the pattern of shear bands and their time of initiation remained unchanged. Results presented herein are for the polynomial equation of state (4).

We note that the values of material parameters given in (2) and (3) may not be valid at strains, strain-rates and temperatures likely to occur in a shear band even though Zhou (1993) used these to analyse a shear band problem. Klepaczko et al. (1987) have stated that most of these material parameters should be taken to depend upon the temperature. In the absence of any test data available to find their temperature dependence, we regard them as constants.

We assume that the block is initially at rest, is stress free and is at a uniform temperature $T_0$. We take all bounding surfaces of the block to be thermally insulated; the vertical edges of the block to be traction free; the top and bottom horizontal edges to be free of tangential tractions and with a vertical component of velocity prescribed on them. The magnitude of this prescribed velocity increases from zero to $50 \text{ m s}^{-1}$ in $5 \mu$s and then stays fixed at $50 \text{ m s}^{-1}$ (cf. Fig. 1(b)); thus, the nominal strain-rate increases from 0 to $5000 \text{ s}^{-1}$ in $5 \mu$s and then stays fixed at $5000 \text{ s}^{-1}$. 
Because of the symmetry of the specimen, and initial and boundary conditions about the horizontal and vertical centroidal axes, we assume that thermomechanical deformations of the block are symmetrical about these two axes and analyse deformations of the material in the first quadrant. Thus conditions arising from the symmetry of deformations, i.e. zero tangential tractions, vanishing normal component of velocity, and zero heat flux are prescribed on these edges.

III. COMPUTATION AND DISCUSSION OF RESULTS

The coupled thermomechanical problem formulated above is highly nonlinear and cannot be solved analytically, therefore, we seek its approximate solution by the finite element method by using the explicit large scale finite element code DYNA2D developed by Whirley et al. (1992). The material subroutine in the code for the Johnson–Cook model was modified to incorporate the thermal softening function included in eqn (1). The code uses 4-noded isoparametric quadrilateral elements with one-point integration rule and an hour-glass control algorithm to suppress the spurious modes. The time step is adjusted adaptively to satisfy the Courant condition after each time increment. For the two phase material being studied here, the speed of the expansion (volumetric) wave equals 4.06 mm $\mu s^{-1}$ in W and 4.69 mm $\mu s^{-1}$ in Fe–Ni–W. Since the volumetric wave travels faster than the shear wave, the time step size is set equal to a fraction of the time taken for the volumetric wave to travel through the smallest Fe–Ni–W element in the mesh. The initial mesh consists of $10^4$ uniform rectangular elements with 100 elements in each direction. However, Fe–Ni–W particles are expected to deform more severely, because of their low yield stress, than pure tungsten particles and will control the time step size. The computed results show that this is not necessarily the case.

Figure 2 depicts, in the reference configuration, three random distributions of 200 Fe–Ni–W particles; the larger rectangles depict two abutting Fe–Ni–W particles. As mentioned earlier, for a fixed number of Fe–Ni–W particles, seven different random distributions, generated by using different seed numbers, were considered. However, the computed time histories of the compressive load were essentially similar for the seven cases. In order to save space, the time histories of the compressive load for 1 cm thick block and for three random distributions of 200 Fe–Ni–W particles are illustrated in Fig. 3. It is clear that, in the beginning, different distributions of 200 Fe–Ni–W particles have a minimal effect on the load required to deform the block. For average axial strains exceeding 0.1, one starts to see differences in the load mainly because around this time the deformation begins to localize. Since the load has been obtained by integrating normal tractions on the top surface, and the integration has a smoothening effect, essentially identical load distributions may give quite different deformation patterns as evinced by the deformed meshes plotted in Fig. 4. Even though almost four dominant narrow regions of intense plastic deformation stand out clearly, there are several thin regions of large plastic deformation. Similar deformation patterns were obtained for other random distributions of 200 Fe–Ni–W particles and also for 10, 20, 400, 600, 800 and 1000 Fe–Ni–W particles. Once the elements have distorted severely, the deforming region needs to be rezoned to obtain satisfactory results. This was not done and it explains why the load histories for the composite block are plotted only up to an average axial strain of 0.15. A comparison of deformed meshes of Fig. 4 with that shown in Fig. 5(b) of Batra and Peng (1995) suggests that the different thermal softening function in eqn (1) and the substitution of weak elements by Fe–Ni–W
Fig. 2. Three random distributions of 200 Fe–Ni–W particles among 10,000 particles of the WHA.

Fig. 3. Time histories of the compressive load (shown positive) required to deform the block for the three random distributions shown in Fig. 2.
particles have no effect on the general orientation of the localized regions of intense plastic deformation or shear bands. Batra and Peng (1995) pointed out that the shear modulus affects this orientation. We note that, except for the two differences mentioned above, the problem studied by Batra and Peng (1995) is identical to the present one. They seemed to believe that a shear band, as indicated by the deformed mesh, formed at an average axial strain of 0.2. They did not study details of the deformation within the localized region; we discuss these below. Because of the different elastic moduli and mass densities of tungsten and Fe–Ni–W particles, the coherence conditions require that both elastic and plastic waves be scattered from Fe–Ni–W particles; no such scattering of elastic waves occurs from the weak elements considered by Batra and Peng.

Fig. 4. Deformed meshes at an average axial strain of 0.1375 for the three random distributions shown in Fig. 2.
For the one-dimensional simple shearing problem Wright and Walter (1987) pointed out that the shear stress within the band drops catastrophically; this was confirmed experimentally by Marchand and Duffy (1988) in torsional deformations of a thin-walled tube. Batra and Zhang (1994) analysed torsional deformations of a thin-walled tube as a three-dimensional problem by using DYNA3D and the Johnson–Cook material model and found that the torque required to twist the tube began to drop rapidly essentially at the same time as the shear band, evidenced by severe distortions of the mesh, initiated. The sudden drop in the torque was clearly demarcated by a sharp change in the slope of the torque versus average shear strain curve. Partly because of the oscillations in the load-average strain curve, it is hard to delineate precisely when the load begins to drop rapidly; these oscillations result from the interaction of waves reflected from the boundaries. For the present problem the time when a shear band initiates cannot be objectively defined either from the load versus average strain curve or from the deformed meshes.

From the deformed meshes shown in Fig. 4 it is hard to ascertain if all of the elements within the severely deformed region are equally deformed. Figure 5 depicts, for five different values of the average strain, the distribution of the effective plastic strain at points on the line joining A(0.4725, 0.4350) and B(0.2275, 0.7450) in the reference configuration. These points were identified visually from the deformed mesh to lie within the severely deformed region. During the deformations of the block the orientation of the line changes from an initial value of $-51.7^\circ$ to $-46.2^\circ$ at an average axial strain of 0.135 when a narrow region of intense plastic strain has developed (cf. Fig. 6). We add that the line $AB$ may pass through different material particles at various times. For the plane strain compression problem being studied here the maximum shear stress should occur in the current configuration on a plane inclined at $\pm 45^\circ$ direction to the horizontal axis. One possible reason for the slight deviation of the angle of line $AB$ from this value is that the cross-section is not necessarily rectangular due to the nonhomogeneous deformations of the composite body. The line passed through only one Fe–Ni–W particle for each one of the

![Fig. 5. Distribution, for five different values of the average axial strain, of the effective plastic strain at points on line $AB$ lying within the severely deformed region.](image-url)
five values of time considered; the location of this point is marked by a dark square on Fig. 5. If a shear band is taken to have fully developed when the effective plastic strain in it equals at least 1.1, then a shear band forms at an average strain of 0.1225. It is clear that the time of formation of a shear band depends upon its definition. The distribution of the effective plastic strain at points on line $AB$ vividly illustrates that all of these points are not equally deformed. Also the plastic strain at any one point is not significantly higher than that at other points on the line; interestingly enough, a tungsten particle has the largest value of the effective plastic strain. Thus one can not determine whether a shear band initiates at one point first and then propagates in either direction along line $AB$ or several shear bands initiate more or less simultaneously at numerous points and then merge together. In order to shed some light on this we have plotted in Fig. 7(a) the time history of the effective stress at points $A$, $B$ and an arbitrarily chosen point $E(0.3375, 0.6050)$ on line $AB$; the effective stress versus effective strain curves for these points are given in Fig. 7(b). Since the effective stress after having attained its peak value drops rather gradually in time (cf. Fig. 7(a)), we adopt the criterion proposed by Batra and Kim (1992) that a shear band initiates in earnest when the effective stress has dropped to 90% of its peak value. Deltort (1994) has postulated that the severe localization of deformation occurs when the shear stress in a simple shearing problem has dropped to 80% of its maximum value. In torsional experiments Marchand and Duffy (1988) associated the initiation of a shear band with the instant where the shear stress begins to drop sharply. According to Batra and Kim’s criterion a shear band initiates at point $B$ first and propagates to points $E$ and $A$; its average speed from $B$ to $E$ equals 802 m s$^{-1}$ and that from $E$ to $A$ 1938 m s$^{-1}$. In general, it is nearly impossible to determine a priori, except possibly when there are one or two relatively strong defects present in the body, where a shear band will initiate first. Batra (1996) has reviewed some of the known results for the shear band speed. Depending upon the definition of a shear band and its direction of propagation, the speed in 4340
steel can vary from $60 \text{ m s}^{-1}$ to $600 \text{ m s}^{-1}$. For example, during the torsional deformations of a thick-walled 4340 steel tube, a shear band, defined as the region wherein the effective plastic strain exceeds 0.7, propagates in the circumferential direction at about $600 \text{ m s}^{-1}$ but at only $60 \text{ m s}^{-1}$ in the radial direction.

Fig. 7. (a) Time history of the effective stress at points $A(0.4725, 0.4350)$, $B(0.2275, 0.7450)$ and an arbitrarily chosen point $E(0.3375, 0.6050)$ on line $AB$. (b) Effective stress versus effective strain curves at points $A$, $B$ and $E$. 

Figure 8(a) and (b) illustrate the distribution of the effective plastic strain and the nondimensional temperature, $T/T_0$, at points on line $CD$ perpendicular to the instantaneous position of $AB$ and passing through the arbitrarily chosen point $E$; co-ordinates of points $C$ and $D$ in the reference configuration are $(0.1592, 4.500)$ and $(0.5336, 0.7600)$ respectively. For the three values of the average strain considered, no Fe–Ni–W particle

![Effective Strain](image1)

(a)

![Temperature Ratio](image2)

(b)

Fig. 8. Distribution, at three values of the average axial strain, of (a) the effective plastic strain and (b) the nondimensional temperature on line $CD$ perpendicular to $AB$. 
occupied a place on line $CD$. It is clear that tungsten particles are intensely deformed, and the maximum temperature of any tungsten particle on line $CD$ equals 1478 K. Of course, this value and the width, approximately $400 \, \mu m$, of the severely deformed region as well as other results presented herein are mesh dependent. A much finer mesh could not be used with the available computational resources. Our experience with adaptively refined meshes (e.g. see Batra and Ko (1992)) indicates that present results are qualitatively correct and a finer mesh may alter slightly the nominal strain at which a shear band initiates. One way to obtain mesh independent results is to use a higher-order gradient theory (Batra and Hwang, 1994; Batra, 1987).

We have plotted in Fig. 9(a) and (b) the velocity of Fe–Ni–W particles for the same value of the average strain at which deformed meshes are plotted in Fig. 4(a) and (b). The velocity of tungsten particles is not shown to avoid cluttering the plots. It is clear that for the deformed mesh of Fig. 4(a), the deforming region is divided into three parts; the material near the top left corner lying above the shear band is moving vertically downward, that between the two localized regions of intense deformation and the right boundary is moving with essentially a uniform velocity in the $-45^\circ$ direction, and the third part between the $-45^\circ$ band and the bottom left corner is practically at rest. Thus in this case, there is a large dead zone formed. For the deformed mesh of Fig. 4(b), the deforming region is divided into two parts; the one above the band moves as a rigid body in the $-45^\circ$ direction and that below the band is virtually at rest. At the instant the deformed mesh in Fig. 4(c) is plotted, the body is divided into four regions; the one near the top left corner is moving vertically downwards with the applied velocity, that near the top right corner is moving with a uniform velocity in the $-45^\circ$ direction, the one enclosed

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Fig. 9. Velocity distribution in Fe–Ni–W particles at an average axial strain of 0.1375 for the random distributions shown in Fig. 2(a) and (c).
by narrow regions of intense deformation on three sides with the fourth side traction free is also moving in the \(-45^\circ\) direction but with a considerably lower speed than the region adjoining the top right corner. The large region bounded by a part of the left edge, bottom edge, a shear band and a part of the right edge constitutes the dead zone.

The effect of the volume percentage of Fe–Ni–W particles on the load versus average strain curve is exhibited in Figs 10(a) and (b). Initially, the load decreases monotonically with an increase in the volume percentage of Fe–Ni–W particles. However, for an average axial strain exceeding 0.10, the load required to deform the composite block with 0.1% Fe–Ni–W particles is less than that for 0.2% Fe–Ni–W particles. Similarly, the load for 4% Fe–Ni–W particles is less than that for 6% Fe–Ni–W particles. This is because the two distributions of Fe–Ni–W particles result in different deformation patterns. For the pure tungsten case (Fig. 10(c)), the load drops at a much higher value of the average strain than that for the case of only one randomly situated Fe–Ni–W particle. However, for two randomly located Fe–Ni–W particles in the lower one-fourth of the block, the load dropped at almost the same time as for pure tungsten. In this case three parallel shear bands, as shown in Figs 11(a) and (b), initiated first in the approximately \(45^\circ\) direction near the bottom right edge but eventually a dominant shear band (cf. Fig. 11(c)) passing through a point near the bottom right corner and inclined at nearly \(-45^\circ\) formed. None of these bands passed through either one of the two Fe–Ni–W particles. This does not support Backmann and Finnegan's (1973) suggestion that shear bands initiate from defects, second phase particles or other impurities in the body. For the block made of pure tungsten, the nonhomogeneity in the deformation occurs because of the interaction between incident waves and waves reflected from the boundaries; for other case waves scattered from Fe–Ni–W particles interact with other waves mentioned earlier. As is obvious from the deformed meshes of Fig. 4 and the random distributions of Fe–Ni–W particles shown in Fig. 3, a shear band does not initiate from every point where the effective plastic strain is expected to be higher than that at its neighboring points. One will speculate that a larger volume percentage of Fe–Ni–W particles will result in an earlier onset of the localization process; however, this can not be concluded from the load versus average strain curves except when there is only one Fe–Ni–W particle. For torsional deformations of a thin-walled tube with thickness varying sinusoidally in the axial direction, Batra et al. (1996) found that the average strain at the initiation of a shear band decreased monotonically with a decrease in the minimum thickness of the tube or an increase in the defect size; Chi (1990), Murphy (1990) and Deltort (1994) obtained similar results experimentally. Molinari and Clifton (1987) and Wright (1994) have investigated analytically the effect of the defect size on the localization of the deformation in a simple shearing problem. However, for the plane strain problem studied by Batra and Peng (1995), and for the present problem the load required to deform the tube did not decrease monotonically with an increase in the number of randomly distributed weak elements. One may argue that a larger number of Fe–Ni–W particles does not necessarily imply that the defect size has increased.

Figure 12 depicts the deformed mesh at an average axial strain of 0.135 when there is only one (0.01%) randomly located Fe–Ni–W particle; in this case two shear bands in the form of an \(X\) pass through the Fe–Ni–W particle. This deformation pattern resembles that observed by Batra and Liu (1989) during plane strain deformations of a steel block with only one weak element at the centroid but differs from the ones computed herein with two or more Fe–Ni–W particles (cf. Figs 4 and 11).
Fig. 10. Compressive load versus average axial strain curves for different volume percentages of Fe-Ni-W particles; in each case these particles are randomly distributed.
We also simulated axisymmetric deformations of a 5-cm long WHA cylinder of 1-cm radius and deformed by prescribing the axial component of velocity on the top and bottom faces. A plane passing through the axis of the cylinder was divided into $10^4$ uniform rectangular elements with 100 elements each in the radial and axial directions. For up to 10% randomly distributed Fe–Ni–W elements on this cross-section and an average axial strain of 50% no shear bands were observed. However, when the exponent $m$ in the thermal softening expression in eqn (1) was changed to 1.0, shear bands formed even in pure tungsten cylinders. This signifies that the thermal softening function plays a critical role especially during the axisymmetric deformations of the body. Walter (1992) has elucidated upon how different thermal softening functions influence the localization of deformation in simple shearing deformations of a thermoviscoelastic body.

Fig. 11. Deformed meshes at average axial strains of (a) 0.1875, (b) 0.2125 and (c) 0.2250 for the case of two randomly located Fe–Ni–W particles.
IV. CONCLUSIONS

We have studied dynamic plane strain and axisymmetric deformations of a WHA with tungsten and iron–nickel–tungsten modeled as thermoviscoplastic materials. Except when the thermal softening effect is made arbitrarily large, no shear bands were observed in axisymmetric deformations. For values of material parameters determined from test findings of Zhou (1983), and for plane strain compression of a WHA block, different volume percentages of Fe–Ni–W particles resulted in essentially identical compressive load versus average axial strain curves for up to an average axial strain of 0.10; for average axial strains exceeding 0.10, the decrease in the load was not a monotonic function of the increase in the volume percentage of Fe–Ni–W particles. Also, for a fixed percentage of Fe–Ni–W particles, no noticeable differences were observed in the load-average axial strain curves for seven different random distributions of 2, 4, 6, 8 and 10% Fe–Ni–W particles. However, in each case, the computed deformation patterns, the number of shear bands, and the sizes and shapes of the dead zones were different. Except for the case when there is only one Fe–Ni–W particle, it is hard to ascertain where a shear band will initiate first. For one random distribution of 2 Fe–Ni–W particles, three parallel shear bands, as evidenced by the deformed mesh, initiated first in the 45° direction but eventually one dominant band formed in the −45° direction; none of these bands passed through either one of the two Fe–Ni–W particles. In every case considered, more than one shear band as characterized by narrow regions of intense plastic deformation formed; the plastic strain distribution in these narrow regions is not uniform. Even though the effective stress at a material point within a shear band when plotted against the effective strain there drops rapidly, the time-rate of drop of the effective stress is low for sometime and picks up subsequently. Also, the compressive load required to deform the block does not drop catastrophically even when a shear band, as indicated by the severely deformed mesh, has fully developed.

Fig. 12. The deformed mesh at an average axial strain of 0.1375 for one randomly located Fe–Ni–W particle.
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REFERENCES


